Numeric Response Questions

Parabola

Q.1 The circle $x^2 + y^t = 5$ meets the parabola $y^t = 4x$ at P and Q, then find the length of PQ.

Q.2 If $\mathrm{P}(\mathrm{at}_1^2, 2at_1)$ and $\mathrm{Q}(\mathrm{at}_2^2, 2at_2)$ are two variable points on the curve $y^2 = 4ax$ and PQ subtends a right angle at the vertex then find the value of $t_1 t_2$.

Q.3 If one end of a focal chord of parabola $y^2 = 16x$ is $(1, -4)$. Then find the length of the focal chord.

Q.4 A focal chord of the parabola $y^2 = 16x$ is tangent to the circle $(x-6)^2 + y^2 = 2$ then find sum of possible values of slope of this chord.

Q.5 The line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$. Then find the product of values of k.

Q.6 The equation of latus rectum of a parabola is $x + y = 8$ and the equation of the tangent at the vertex is $x + y = 12$, if the length of the latus rectum is $k\sqrt{2}$ then find k.

Q.7 Find the latus rectum of the parabola $y^2 = 4ax$ whose focal is PSQ such that $\mathrm{SP} = 3\&\mathrm{SQ} = 2$,

Q.8 An equilateral triangle is inscribed in the parabola $y^2 = 4ax$, whose vertex is at the vertex of the parabola. If the length of its side is $ka\sqrt{3}$ then find k.

Q.9 If the distance between directrix and latus rectum of a parabola is 4 units, then find the length of its latus rectum.

Q.10 Find the length of latus rectum of the parabola $5y^2 - 3x - 8y - 1 = 0$.

Q.11 If the line $y = 3x + 1$ touches parabola $y^2 = kx$, then find k.

Q.12 An equilateral triangle is inscribed in the parabola $y^2 = 4ax$ with one vertex at the origin. The radius of the circum circle of that triangle is ka then find k.

Q.13 The locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with the directrix $x = \underline{\lambda}$, then find λ.

Q.14 Find the length of chord of the parabola $y^4 = 4x$ which passes through the vertex and inclined at an angle $60'$ with x-axis,

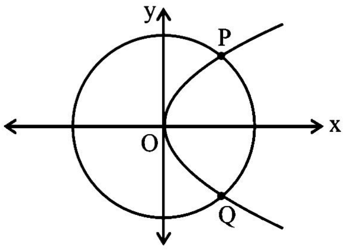
Q.15 Find the number of real tangents that can be drawn to the curve $y^2 + 2xy + x^2 + 2x + 3y + 1 = 0$ from the point $(1, -2)$.

ANSWER KEY

1. 4.00	**2.** – 4.00	**3.** 25.00	**4.** 0.00	**5.** –32.00	**6.** 8.00	**7.** 4.80
8. 8.00	**9.** 8.00	**10.** 0.60	**11.** 12.00	**12.** 8.00	**13.** 0.00	**14.** 2.67
15. 0.00						

Hints & Solutions

1.



Circle $x^2 + y^2 = 5$ ...(i)

Parabola $y^2 = 4x$ ...(ii)

solving (i) & (ii)

$x^2 + 4x = 5$

$\Rightarrow x^2 + 4x - 5 = 0$

$\Rightarrow (x + 5)(x - 1) = 0$

$\Rightarrow x = -5, x = 1$

Taking x = 1

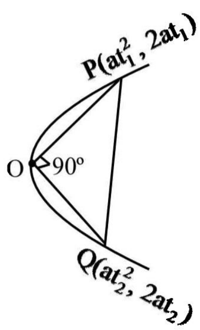
$y^2 = 4$

$y = \pm 2$

P(1, 2), Q(1, –2)

PQ = 4

2.



Slope of OP, $m_1 = \dfrac{2at_1 - 0}{at_1^2 - 0}$

$m_1 = \dfrac{2}{t_1}$

Slope of OQ $m_2 = \dfrac{2}{t_2}$

$m_1 m_2 = -1$

$\Rightarrow \dfrac{2}{t_1} \cdot \dfrac{2}{t_2} = -1$

$\Rightarrow t_1 t_2 = -4$

3. $y^2 = 16x$

a = 4

Let $(at^2, 2at) = (1, -4)$

$\Rightarrow 2at = -4 \Rightarrow 8t = -4 \Rightarrow t = -1/2$

$\Rightarrow \text{Length} = a\left(t + \dfrac{1}{t}\right)^2$

$= 4\left(-\dfrac{1}{2} - 2\right)^2 = 25$

4. $y^2 = 16x$, a = 4

Focus (4, 0)

Let line passing through (4, 0) is

$y - 0 = m(x - 4)$

$\Rightarrow mx - y - 4m = 0$

Centre of circle (6, 0), $r = \sqrt{2}$

p = r

$\dfrac{|6m - 0 - 4m|}{\sqrt{m^2 + 1}} = \sqrt{2}$

$\Rightarrow 4m^2 = 2(m^2 + 1)$

$\Rightarrow m^2 = 1$

5. $y^2 - kx + 8 = 0$

$\Rightarrow y^2 = kx - 8$

$\Rightarrow \boxed{y^2 = k\left(x - \dfrac{8}{k}\right)}$

$Y^2 = 4aX$

Directrix : X = –a

$\Rightarrow x - \frac{8}{k} = \frac{-k}{4}$

$\Rightarrow x = \frac{8}{k} - \frac{k}{4}$ ...(1)

Given directrix x = 1 ...(2)

$\therefore \frac{8}{k} - \frac{k}{4} = 1$

$\Rightarrow k = -8, 4$

6. $a = \left|\frac{-8+12}{\sqrt{1+1}}\right| = \frac{4}{\sqrt{2}}$

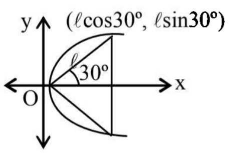
Length of latusrectum $= 4a = 4\left(\frac{4}{\sqrt{2}}\right)$

$= 8\sqrt{2}$

7. $2a = \frac{2\,SP \times SQ}{SP + SQ} = \frac{2 \times 3 \times 2}{3+2} = \frac{12}{5}$

$\therefore$ latus rectum = 24/5

8.



Point at a 'ℓ' unit distance from origin will be

$\left(\frac{\ell\sqrt{3}}{2}, \frac{\ell}{2}\right)$

It lies on parabola, so

$\frac{\ell^2}{4} = 4a\,\frac{\ell\sqrt{3}}{2} \Rightarrow \ell = 8a\sqrt{3}$

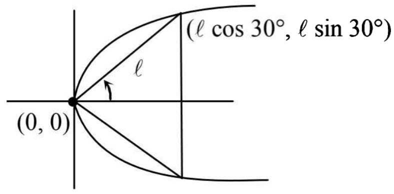
9. If the distance between the directrix and latus rectum is 2a, then length of latus rectum is 4a. Therefore, length of latus rectum is 8 units.

10. Parabola $5y^2 - 3x - 8y - 1 = 0$

Length of L.R $= \left|\frac{-3}{5}\right| = \frac{3}{5}$

11. $c = \frac{a}{m} \Rightarrow 1 = \frac{k/4}{3} \Rightarrow k = 12.$

12.



As $\left(\frac{\ell}{2}\right)^2 = 4a.\,\ell\frac{\sqrt{3}}{2}$

$\Rightarrow \frac{\ell}{4} = \frac{4a\sqrt{3}}{2} \Rightarrow \ell = 8a\sqrt{3}$

Radius $= \frac{2}{3} \times 12a = 8a.$

13. $(at^2, 2at), (a, 0)$

$h = \frac{at^2 + a}{2}, k = \frac{2at + 0}{2}$

$\therefore 2h - a = at^2, t = \frac{k}{a}$

$\therefore 2h - a = a\left(\frac{k}{a}\right)^2$

$\Rightarrow k^2 = 2ah - a^2$

$\Rightarrow y^2 = a(2x - a)$

Whose directrix is x = 0.

14. $y^2 = 4x$

$\{m = \tan 60^\circ = \sqrt{3}, c = 0, a = 1\}$

length PQ $= \frac{4}{m^2}\sqrt{a(1+m^2)(a-cm)}$

$= \frac{4}{3}\sqrt{1(1+3)(1-0)} = \frac{8}{3}$

15. $S_1 = 4 - 8 + 1 + 2 - 6 + 1$

$S_1 = -6$

$S_1 < 0$ (inside the parabola)

Zero tangent